**ICT 2105: Numerical Analysis**

**Newton-Raphson Method**

Let x0 be an approximate root of the equation f(x) = 0

and

Let x = x0 + h be an exact root so that f(x0 + h) = 0, where h being a small quantity.

Now expanding f(x0 + h) by Taylor’s series, we get –

f(x0 + h) = f(x0) + hf’(x0) + hf’’(x0) + …….

f(x0) + hf’(x0) + f’’(x0) = 0 ∵ x0 + h = 0

Since h is very small, neglecting the second and higher order terms of h, we obtain an approximate value of h say h from the above equation.

f(x0) + h1f’(x0) = 0 ⇒ h1 = - ………. …. (1)

A better approximation than x0 is therefore may lie at x1 where

x1 = x0 + h1

x1 = x0 - ………….. from (1)

Now using x1 in the place of x0 and x2 in the place of x1 we get

Now replace x1 for x0

and x2 for x1

x2 = x1 - , which is little better than before e.g. x1.

Similarly

X3 = x2 - , which is little better than x2.

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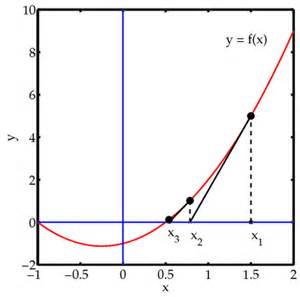
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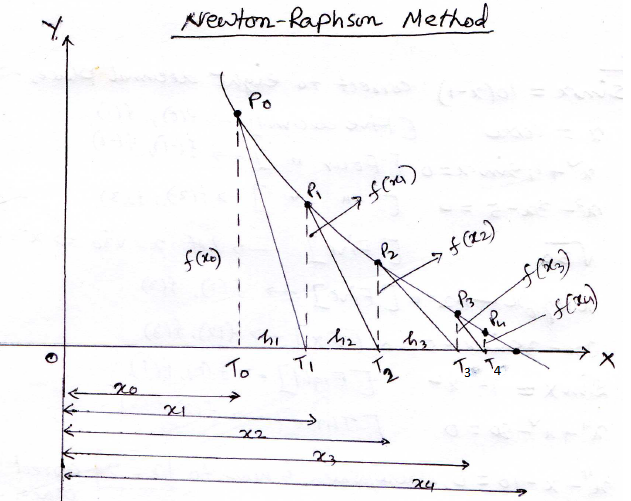
Continuing like this, we iterate this process until

Xn+1 = xn - , where n = 0,1,2,3,………

This method of successive approximation is called the Newton-Raphson Method.

The method can be used for both algebraic and transcendental equations and works for complex equation with complex coefficients.





**Examples of NR(Newton-Rapson Method):**

**Ex-1:** Find the root of by NR method correct to **five** decimal places.

**Solution:** let,

So, at least one root lies between 1 and 2.

Now by NR method, we get:

From (1) putting we get,

Hence, the required root is 1.78377.

**Ex-2:** Find the root of correct to **Five** decimal places by NR method.

**Solution:** let,

Here, f(0)=4>0 and f(1)=-1<0 .

So, at least one root lies between o and 1.

Now, by NR method we get,

Taking the relation (1) gives,

so, the required root is **.**

**Ex-3** Find the root of 3x-cosx-1=0 by Newton-Raphson method correct to **six** decimal places.

**Solution:** Let, f(x) =3x-cosx-1

So, f ’(x) = 3+sinx

Here, f(0) = 0-0-1 <0

And f(1) = 3-cos(1)-1 = 1.45970 >0

So, one root lies between 0 and 1.

Now, by NR method we get:

Taking the relation (1) gives,

So, The required root is: **.**

**Ex-4:** Find the root of by NR.

**Solution:**

Let,

Here, f(0)=1>0 and f(1)=-2.17798<0 so, one root lies between 0 and 1.

By NR method we get,

Taking we get from (1)

So the required root is:

**Exercise for NR method:**

1. correct to eight decimal places ⇒ f(1),f(2)
2. [five decimal] ⇒ f(0), f(1)
3. [Four decimal] ⇒ f(-1), f(-2)
4. [Four decimal] ⇒ f(2), f(3)
5. [Five decimal] ⇒ f(5), f(6)
6. [Five decimal] ⇒ f(2), f(3)
7. [Six decimal] ⇒ f(1), f(2)
8. [Eight decimal] ⇒ f(0), f(1)
9. [Three decimal] ⇒ f(2), f(3)
10. , where/which is near to x=2 [Three decimal] ⇒ f(1), f(2)